1. $U(x) = ax^3 + bx^2 + U_0 \quad (b>0)$

a) $F_u = -U'(x)$

$U'(x) = 3ax^2 + 2bx$

$U'(0) = 0$

$F_u(0) = 0 \rightsquigarrow$ particle is in equilibrium

b)

$$U(x) \approx \underset{U_0}{\underset{\parallel}{U(0)}} + \underset{0}{\underset{\parallel}{U'(0)}}(x) + U''(0)\frac{x^2}{2} \,|\, \cdots$$

$U''(x) = 6ax + 2b \qquad U''(0) = 2b$

$\longrightarrow U(x) \approx U_0 + bx^2$

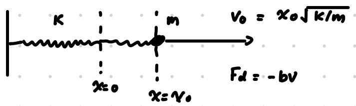
c) $F = -U'(x) \approx -2bx$

$m\ddot{x} = -2bx$

$\ddot{x} = -\frac{2b}{m}x$

$w = \sqrt{2b/m}$

2.



a) $m\ddot{x} = -b\dot{x} - Kx$

$m\ddot{x} + b\dot{x} + Kx = 0$

$\ddot{x} + \gamma\dot{x} + w_0^2 x = 0$

b) $x(t) = A_0 e^{-\gamma t/2}\cos(wt+\phi)$

$\dot{x}(t) = -\frac{\gamma}{2}A_0 e^{-\gamma t/2}\cos(wt+\phi) - wA_0 e^{-\gamma t/2}\sin(wt+\phi)$

(I) $x(0) = A_0\cos\phi = x_0$

(II) $\dot{x}(0) = -\frac{\gamma}{2}A_0\cos\phi - wA_0\sin\phi = x_0\sqrt{\frac{K}{m}} = w_0 x_0$

We assume very light damping $\rightsquigarrow$ $\gamma/2 \ll w_0$

$w = \sqrt{w_0^2 - \gamma^2/4}$

$\approx w_0$

$$A_0\left(-\underset{\substack{\gamma/2 \ll w_0 \\ \text{approx to be zero}}}{\underset{\parallel}{\frac{\gamma/2}{w_0}}}\cos\phi - \underset{\substack{w \approx w_0 \\ \text{approx to be } 1}}{\underset{\parallel}{\frac{w}{w_0}}}\sin\phi\right) = x_0 \Rightarrow -A_0\sin\phi \approx x_0$$

$\Rightarrow$ (II) $A_0\sin\phi \approx -x_0$

$(I)^2 + (II)^2 \quad A_0^2\cos^2\phi + A_0^2\sin^2\phi = A_0^2 = 2x_0^2$

$\frac{(II)}{(I)} \quad \frac{A_0\sin\phi}{A_0\cos\phi} = \tan\phi = -1 \rightsquigarrow \phi \approx -\pi/4$

c) $E_0 = \frac{1}{2}Kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}Kx_0^2 + \frac{1}{2}m x_0^2\frac{K}{m} = Kx_0^2$

$x_0^2 = A_0^2/2$

$E_0 = KA_0^2/2$

3.

$F_d = -bV$

a) $\gamma/2 < w_0$

$$\frac{b}{2m} < w_0 = \sqrt{\frac{2K}{m}}$$

$$b < \sqrt{\frac{2K}{m}} \cdot 2m = \sqrt{\frac{2K \cdot 4m^2}{m}} = \sqrt{8Km}$$

$$b < \sqrt{8Km}$$

b) $m\ddot{x} + b\dot{x} + 2Kx = F_0 \cos wt$

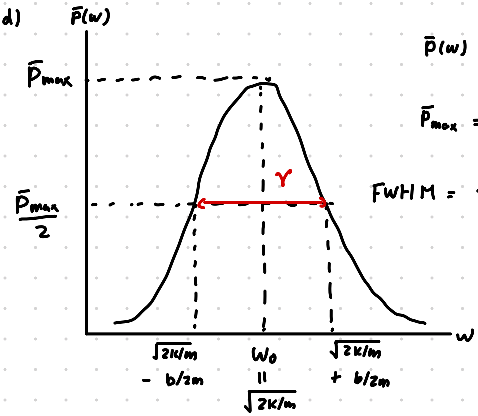
$$\ddot{x} + \gamma\dot{x} + w_0^2 x = (F_0/m)\cos wt$$

c) Steady state : $x(t) = A(w)\cos(wt - \delta)$

$$\rightsquigarrow A(w) = \frac{F_0}{m\sqrt{(w_0^2-w^2)^2 + \gamma w^2}} = \frac{F_0}{m\sqrt{(2K/m - w^2) + (bw/m)^2}}$$

$$\delta(w) = \arctan\left(\frac{\gamma w}{w_0^2 - w^2}\right) = \arctan\left(\frac{bw/m}{2K/m - w^2}\right)$$

d)

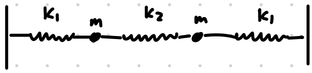


$$\bar{P}(w) = \frac{w^2 F_0^2 \gamma}{2m((w_0^2 - w^2)^2 + w^2\gamma^2)}$$

$$\bar{P}_{max} = \bar{P}(w_0) = \frac{\cancel{w_0^2} F_0^2 \gamma}{2m \cancel{w_0^2} \gamma^2} = \frac{F_0^2}{2m\gamma}$$

FWHM = $\gamma = b/m$

4.



a) $m\ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2) = -(k_1 + k_2)x_1 + k_2 x_2 \quad (I)$

$m\ddot{x}_2 = -k_2(x_2 - x_1) - k_1 x_2 = k_2 x_1 - (k_1 + k_2)x_2 \quad (II)$

b) $q_1 = x_1 + x_2$

$q_2 = x_1 - x_2$

$(I) + (II): \; m(\ddot{x}_1 + \ddot{x}_2) = m\ddot{q}_1 = -k_1 x_1 - k_1 x_2 = -k_1 q_1$

$$\ddot{q}_1 = -\frac{k_1}{m} q_1$$

$(I) - (II): \; m(\ddot{x}_1 - \ddot{x}_2) = m\ddot{q}_2 = -k_1 x_1 - 2k_2 x_1 + k_1 x_2 + 2k_2 x_2$

$$= -(k_1 + 2k_2)(x_1 - x_2)$$

$$= -(k_1 + 2k_2) q_2$$

$$\ddot{q}_2 = -\frac{(k_1 + 2k_2)}{m} q_2$$

general soln to $\ddot{x} = -w^2 x$: $x = A\cos(wt + \underbrace{\theta}_{=0})$

c) $q_1(t) = C_1 \cos(w_1 t) \quad w_1 = \sqrt{k_1/m}$

$q_2(t) = C_2 \cos(w_2 t) \quad w_2 = \sqrt{(k_1 + 2k_2)/m}$

d) $x_1 = \frac{1}{2}(q_1 + q_2) = \frac{1}{2}(C_1\cos(w_1 t) + C_2\cos(w_2 t))$

$x_2 = \frac{1}{2}(q_1 - q_2) = \frac{1}{2}(C_1\cos(w_1 t) - C_2\cos(w_2 t))$

c) w_1 : $C_2 = 0$

$$\left.\begin{aligned} x_1 &= \frac{C_1}{2}\cos w_1 t \\ x_2 &= \frac{C_1}{2}\cos w_1 t \end{aligned}\right\} \text{ Same amplitude \& in phase}$$

w_2 : $C_1 = 0$

$$\left.\begin{aligned} x_1 &= \frac{C_2}{2}\cos w_2 t \\ x_2 &= -\frac{C_2}{2}\cos w_2 t \end{aligned}\right\} \text{ Same amplitude \& phase difference of } \pi$$

(antisymmetric)